Mathematical Notes 2012 vol.92 N3-4, pages 412-421

Integral properties of the classical warping function of a simply connected domain

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Abstract

Let u(z,G) be the classical warping function of a simply connected domain G. We prove that the L p-norms of the warping function with different exponents are related by a sharp isoperimetric inequality, including the functional u(G) = sup x∈Gu(x, G). A particular case of our result is the classical Payne inequality for the torsional rigidity of a domain. On the basis of the warping function, we construct a new physical functional possessing the isoperimetric monotonicity property. For a class of integrals depending on the warping function, we also obtain a priori estimates in terms of the L p-norms of the warping function as well as the functional u(G). In the proof, we use the estimation technique on level lines proposed by Payne. © 2012 Pleiades Publishing, Ltd.

http://dx.doi.org/10.1134/S0001434612090143

Keywords

isoperimetric inequality, isoperimetric monotonicity, level lines, Payne inequality, Schwartz symmetrization, torsional rigidity, warping function